Technical note

Phase distortion in biological signal analysis caused by linear phase FIR filters

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Keywords—*FIR digital filters, Phase distortion, Real-time digital filtering*

Med. & Biol. Eng. & Comput., 1987, **25**, 231–238

1 Introduction

DIGITAL SIGNAL processing techniques have found widespread applications in the biomedical field. In particular digital filtering has been commonly used in many areas of biomedicine such as cardiology (PAN and TOMPKINS, 1985), neurology (FRIDMAN *et al.*, 1982), otology (ENGELKEN *et al.*, 1982), neurophysiology (WHEELER and VALESANO, 1985). Digital filters have been implemented either in hardware (SCHLUTER, 1981) or as software routines for a general-purpose computer (CERUTTI *et al.*, 1985). Hardware implementations are usually intended for real-time operation.

In biomedical applications a special emphasis is given to finite duration impulse response (FIR) filters. They are preferred over the infinite impulse response filters (IIR) because they can be designed to have a linear phase response. This property is specially important in the filtering of the pulse-like signals which are commonplace in biomedicine. However, many linear phase FIR filters reported in the literature are not true linear phase filters because their phase responses exhibit jumps between linear segments. When phase discontinuities occur in FIR filters with a small number of coefficients, distortions in the output waveform may occur. These short duration FIR filters are being used in real-time biomedical signal processing systems such as cardiac arrhythmia monitors (SCHLUTER, 1981; PAN and TOMPKINS, 1985).

This note shows how and when linear phase FIR filters may introduce phase distortions. Two popular FIR filter design methods are extended to cover true linear phase filters. Some examples compare the characteristics and the behaviour of linear phase and true linear phase filters.

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First received 12th February and in final form 7th August 1986

2 Linear phase and true linear phase FIR filters

There are two kinds of linear phase FIR filters: those with an even-symmetric impulse response and those with an odd-symmetric impulse response. The first kind is the most important in biomedical applications and is well suited for the design of low-pass, high-pass, bandpass and bandstop filters. The second kind is characterised by a $\pi/2$ term in the phase response and is therefore more suited to the design of differentiators and Hilbert transformers.

The unit sample response or impulse response of a FIR filter will be denoted $h(L)$, $h(L+1)$, ..., $h(L+N-1)$, where L and N are integers and $h(\cdot)$ is real. N is the duration of the impulse response. Two cases will be discussed: $L = 0$ for a causal system and $L = -(N-1)/2$ (N odd) or $L = -N/2$ (N even) for a noncausal system.

2.1 *Property 1*

(Rabiner and Gold, 1975; Cappellini *et al.*, 1978)

If the impulse response of a causal FIR filter has even symmetry with respect to the abscissa $(N-1)/2$ then its frequency response may be written as:

$$H(e^{j\omega}) = R(e^{j\omega})e^{-j\alpha\omega} \quad (1)$$

where $R(e^{j\omega})$ is a *real* function of ω, usually different from $|H(e^{j\omega})|$,

$$\alpha = (N-1)/2$$

Owing to the apparent phase linearity $-\alpha\omega$, a filter that has a frequency response given by eqn. 1 is called a *linear phase filter*. However, it should be stressed that whenever $R(e^{j\omega})$ changes sign the phase response has a jump discontinuity of size π. This means that the phase response is piecewise linear with as many jumps of π as the number of zero crossings in $R(e^{j\omega})$ (see Fig. 1*a*). This type of discontinuous phase response is quite different from the ideal linear phase characteristics.

A subset of the set of linear phase FIR filters with an even-symmetric impulse response can be defined as follows:

Definition

A causal *true linear phase FIR filter* has a frequency response given by

$$H(e^{j\omega}) = P(e^{j\omega})e^{-j\alpha\omega} \quad (2)$$

where $P(e^{j\omega})$ is a *non-negative* real function of ω with $P(e^{j\omega}) = |H(e^{j\omega})|$

$$\alpha = (N-1)/2$$

The phase response is continuous and linear for $\omega \in (-\pi, \pi)$ (see Fig. 1*b*). Therefore, only true linear phase FIR filters can be guaranteed to introduce no phase distortion in low-pass, high-pass, bandpass and bandstop filters.

The extension of Property 1 to noncausal filters is straightforward and expr. 1 results with $\alpha = 0$ (Cappellini *et al.*, 1978). It should be emphasised that the resulting filter is not, in general, a zero phase FIR filter. Its phase response will switch between zero and π whenever $R(e^{j\omega})$ changes sign (Fig. 1*c*). A zero phase FIR filter is one that obeys expr. 2 with $\alpha = 0$ (Fig. 1*d*).

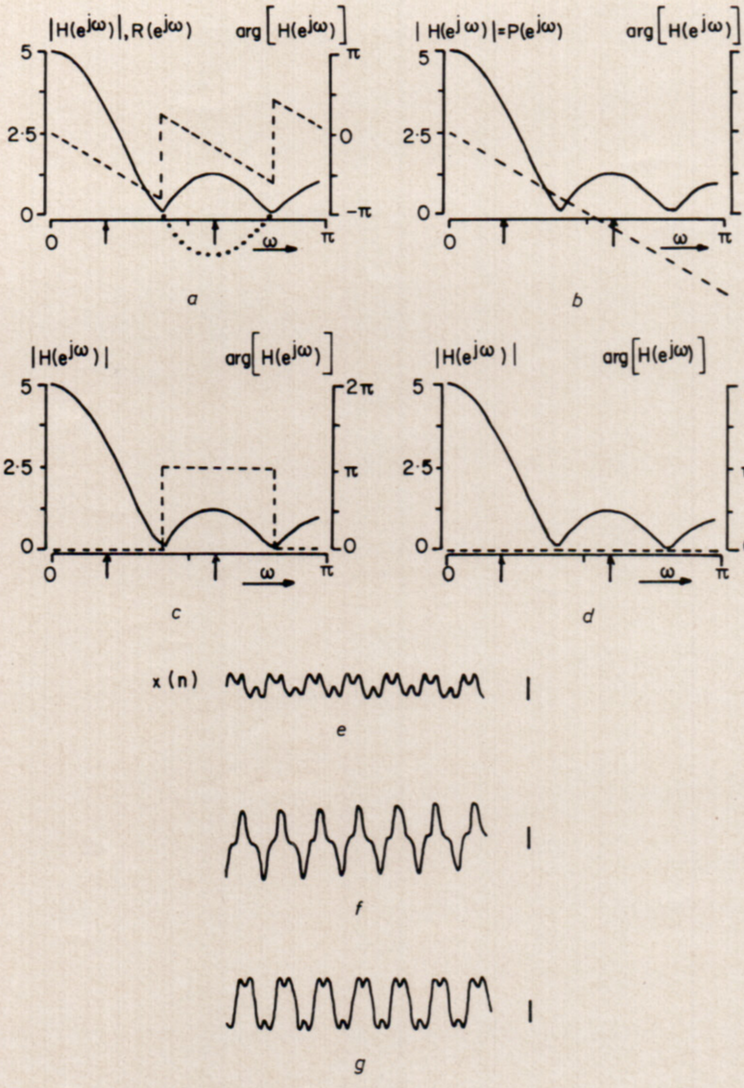

Fig. 1 *(a) Causal linear phase FIR filter with transfer function* $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} = (1 - z^{-5})/(1 - z^{-1})$; *(b) causal true linear phase FIR filter; (c) noncausal linear phase FIR filter with transfer function* $H(z) = z^2 + z + 1 + z^{-1} + z^{-2}$; *(d) noncausal true linear phase FIR filter or zero phase filter; (e) input signal* $x(n) = \sin(2\pi n/10) + \sin(6\pi n/10)$ *to the filters; (f) output signal from the causal linear phase filter; (g) output signal from the causal true linear phase filter. In (a), (b), (c) and (d) the solid line is* $|H(e^{j\omega})|$ *(the same in all figures) and the broken line is the phase response. In (a) the dotted line indicates where* $R(e^{j\omega})$ *is different from* $|H(e^{j\omega})|$. *The two vertical arrows in each graph indicate the frequencies* $2\pi/10$ *and* $6\pi/10$ *of the sinusoids that compose the input signal* $x(n)$. *Calibrations in (e), (f) and (g) are the same.* $H(.) = 0$ *at* $\omega_1 = 4\pi/10$ *and* $\omega_2 = 8\pi/10$

A minor addition to the definition given above is that if $P(e^{j\omega})$ is nonpositive real a true linear phase filter is obtained but with a sign inversion in the output signal. But as this case is equivalent to the non-negative condition only the latter will be considered in what follows.

Next it is shown in what cases phase distortion may occur in linear phase FIR filters. Let us take as a prototype the low-pass linear phase FIR filter whose frequency response $H(e^{j\omega}) = R(e^{j\omega})e^{-j\alpha\omega}$ is depicted in Fig. 1*a*. Its phase response has jumps of size π. The input signal $x(n)$ has a Fourier transform $X(e^{j\omega}) = |X(e^{j\omega})|e^{j\phi_x(\omega)}$. The output signal's Fourier transform is

$$Y(e^{j\omega}) = R(e^{j\omega})|X(e^{j\omega})|e^{-j(\alpha\omega + \phi_x(\omega))} \quad (3)$$

Therefore, the output signal $y(n)$ can be written as:

$$y(n) = \frac{1}{\pi}\int_0^{\pi} R(e^{j\omega})\,|X(e^{j\omega})|\cos(\omega n + \alpha\omega + \phi_x(\omega))\,d\omega$$

$$= \underbrace{\frac{1}{\pi}\int_0^{\omega_1} R(e^{j\omega})\,|X(e^{j\omega})|\cos(\omega n + \alpha\omega + \phi_x(\omega))\,d\omega}_{a(n)}$$

$$+ \underbrace{\frac{1}{\pi}\int_{\omega_1}^{\omega_2} R(e^{j\omega})\,|X(e^{j\omega})|\cos(\omega n + \alpha\omega + \phi_x(\omega))\,d\omega}_{b(n)}$$

$$+ \underbrace{\frac{1}{\pi}\int_{\omega_2}^{\pi} R(e^{j\omega})\,|X(e^{j\omega})|\cos(\omega n + \alpha\omega + \phi_x(\omega))\,d\omega}_{c(n)} \quad (4)$$

Expr. 4 shows that the output sequence is the superposition of the three signals $a(n)$, $b(n)$ and $c(n)$. When compared with the ideal distortionless filter (= true linear phase filter) the sequence $b(n)$ has a sign inversion due to the change of sign that occurs in $R(e^{j\omega})$ in the interval (ω_1, ω_2). From these considerations it can be concluded that distortions in $y(n)$ will occur whenever sequence $b(n)$ has samples with significant amplitudes when compared with the samples from signal $a(n)$. This will happen when the filter has a poorly attenuated stopband and the signal's spectrum has significant components at the stopband frequencies. Filters with a small number of impulse response coefficients have poorly attenuated stopbands and hence may introduce phase distortions at the output (see Section 4).

It was shown above that if $R(e^{j\omega})$ has a sign inversion then phase distortions may occur. These sign inversions in $R(e^{j\omega})$ may occur in filters whose ideal amplitude response is made up of at least one stopband with zero gain (or minus infinity if gain is measured in dB). Typical examples are the low-pass, high-pass, bandpass and bandstop filters. FIR filters with odd-symmetric impulse responses are most useful for differentiators and Hilbert transformers whose ideal amplitude responses do not have any stopband with null gain. Hence their phase responses will not exhibit any undesirable phase switchings. That is why the emphasis in this work is given to FIR filters with even-symmetric impulse responses.

The general conclusion is that whenever an application requires few impulse response coefficients, for example due to speed or cost requirements, a true linear phase filter should be used to avoid possibly harmful phase distortions.

3 Design of true linear phase filters

In this section some guidelines will be presented for the design of true linear phase filters. Two methods described in the literature for the design of linear phase FIR filters will be extended to encompass true linear phase filters. Section 3.1 presents the integer coefficient design of LYNN (1977) and Section 3.2 presents the window method.

Two other popular design methods are frequency sampling with unconstrained samples in the transition band and the minimax (RABINER and GOLD, 1975). If a true linear phase filter is to be designed using either of these two methods a linear programming approach seems advisable as it is easy to impose linear constraints at the stopbands to obtain $R(e^{j\omega}) \geqslant 0$.

3.1 *Design of integer coefficient realisations by pole-zero placement*

LYNN (1977) presented a very useful method for designing linear phase FIR filters with integer coefficients. The method has been employed, for example, by AHLSTROM and TOMPKINS (1985) in the design of low-pass and high-pass FIR filters for the real-time processing of ECG signals. It consists of placing zeros equally spaced around the unit circle and then choosing poles to cancel zeros at convenient places. For low-pass filters the following transfer function is used:

$$H(z) = \frac{1 - z^{-m}}{1 - z^{-1}}, \ m \in Z^+$$

It is easy to show that there is a pole-zero cancellation at $z = 1$. The filter is realised by the recursive difference equation with integer coefficients $y(n) = y(n-1) + u(n) - u(n-m)$, which is very suitable for fast operation.

For high-pass filters Lynn suggests using

$$H(z) = \frac{1 - z^{-m}}{1 + z^{-1}}, \ m \in Z^+$$

However, a constraint is needed: m should be even. Only in this case is the pole at $z = -1$ cancelled by a zero, resulting in an adequate frequency response. Another important observation is that, for any m, the phase response of $H(z)$ has an undesired constant $\pi/2$ term that causes distortions at the output. If there is no DC level to be filtered out, one should use instead

$$H(z) = \frac{1 + z^{-m}}{1 + z^{-1}}, \ m \text{ odd}$$

which has the desired phase response $-\omega(m-1)/2$ for small ω.

For bandpass filters the following transfer function is used:

$$H(z) = \frac{1 - z^{-m}}{1 - 2\cos\theta\, z^{-1} + z^{-2}}, \ m \in Z^+$$

where a complex conjugate pole pair $z = e^{\pm j\theta}$ cancels a corresponding complex zero pair. The sampling rate can be adjusted to guarantee that $2\cos\theta$ is an integer. For applications where the sidelobes of the amplitude response need to be smaller, Lynn suggests using second- or higher-order zeros and poles. The filters designed using the above method may or may not have a true linear phase response. For example, the bandpass filter $(1 - z^{-24})/(1 - z^1 + z^{-2})$ in LYNN (1977) is not of the true linear phase type (see Fig. 5*b*).

A simple way of assuring true linear phase and integer coefficients is to raise the numerator and denominator of the transfer functions proposed by Lynn to an *even* power. To see why this works it is sufficient to raise eqn. 1 to an even power $2k$ ($k \in Z^+$), obtaining $R^{2k}(e^{j\omega})e^{-2jk\alpha\omega}$ where $R^{2k}(e^{j\omega})$ is a non-negative function. The new filter transfer functions become:

(i) low-pass filters

$$H(z) = \frac{(1 - z^{-m})^{2k}}{(1 - z^{-1})^{2k}}, \ k \in Z^+$$

(ii) high-pass filters

$$H(z) = \frac{(1 - z^{-m})^{2k}}{(1 + z^{-1})^{2k}}, \ m \text{ even}, \ k \in Z^+$$

(iii) bandpass filters

$$H(z) = \frac{(1 - z^{-m})^{2k}}{(1 - 2\cos\theta\, z^{-1} + z^{-2})^{2k}},\ k \in Z^+$$

It should be noted that the phase response in case (ii) will either have a constant π term (for odd k) or a 2π term (for even k), neither of which causes any phase distortions. For odd k there is a sign inversion at the filter's output. The cost paid for the improvement in phase (and magnitude) response is a usually small increase in computation time.

3.2 *Design using windows*

In this well known method an ideal infinite duration impulse response is truncated to N samples by a window. The ideal impulse response is usually from an ideal low-pass, high-pass, bandpass or bandstop filter. Less used ideal filters are the differentiators and Hilbert transformers. There is a wide variety of windows, such as the rectangular, triangular, Hann, Hamming, Kaiser (HARRIS, 1978). Two approaches will be presented for the window design of true linear phase FIR filters.

3.2.1 *Use of windows with non-negative Fourier transforms* As the filter's finite duration impulse response $h(n)$ is obtained by the multiplication of the ideal impulse response $h_d(n)$ with the window sequence $w(n)$, its frequency response $H(e^{j\omega})$ is the periodic convolution of $H_d(e^{j\omega})$ with $W(e^{j\omega})$ (OPPENHEIM and SCHAFER, 1975). $H_d(e^{j\omega})$ is the ideal or desired frequency response and $W(e^{j\omega})$ is the Fourier transform of the window sequence. For causal linear phase FIR filters of length N the following relationships hold:

$$H_d(e^{j\omega}) = R_d(e^{j\omega})e^{-j\omega\alpha}$$

$$W(e^{j\omega}) = R_w(e^{j\omega})e^{-j\omega\alpha}$$

with $\alpha = (N-1)/2$. Therefore

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j(\omega-\theta)})W(e^{j\theta})\,d\theta$$

Substituting the expressions for $H_d(e^{j\omega})$ and $W(e^{j\omega})$ into the expression above we have

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} R_d(e^{j(\omega-\theta)})R_w(e^{j\theta})e^{-j(\omega-\theta)\alpha}e^{-j\theta\alpha}\,d\theta$$

and therefore

$$H(e^{j\omega}) = \frac{e^{-j\omega\alpha}}{2\pi}\int_{-\pi}^{\pi} R_d(e^{j(\omega-\theta)})R_w(e^{j\theta})\,d\theta \qquad (5)$$

Expr. 5 shows that if $R_d(\cdot)$ and $R_w(\cdot)$ are both non-negative then $H(e^{j\omega}) = P(e^{j\omega})e^{-j\omega\alpha}$ where $P(e^{j\omega})$ is non-negative. The typical ideal functions $R_d(\cdot)$ are non-negative, made of plateaus at positive or zero values for low-pass, high-pass, bandpass and bandstop filters. Therefore a sufficient condition for the design of true linear phase filters is that the window should have a non-negative Fourier transform, meaning that $R(e^{j\omega}) \geqslant 0$, $\omega \in [0, \pi]$. Some known windows satisfy this property, including the triangular (or Bartlett), Parzen, Bohman (or Papoulis) (HARRIS, 1978). They provide lower sidelobes than the rectangular window but they widen the transition band.

3.2.2 *Use of arbitrary windows with adjustment of a single impulse response coefficient* The general expressions for $R(e^{j\omega})$ when $h(n)$ is even-symmetric (CAPPELLINI *et al.*, 1978; RABINER and GOLD, 1975) indicate that for odd N there is a simple way of imposing non-negativeness on $R(e^{j\omega})$. From the above-mentioned references

$$R(e^{j\omega}) = a(0) + \sum_{m=1}^{(N-1)/2} a(m)\cos(\omega m), \quad N \text{ odd} \qquad (6)$$

where, for a causal filter

$$a(n) = \begin{cases} h\left(\dfrac{N-1}{2}\right) & n = 0 \\ 2h\left(\dfrac{N-1}{2} - n\right) & n = 1, \ldots, \dfrac{N-1}{2} \end{cases}$$

and for a noncausal filter

$$a(n) = \begin{cases} h(0) & n = 0 \\ 2h(n) & n = 1, \ldots, \dfrac{N-1}{2} \end{cases}$$

The simple design procedure proposed consists of choosing an odd N and designing the filter using any arbitrary window (e.g. rectangular, Hann, Hamming). At this step a set of coefficients $a(0), a(1), \ldots, a((N-1)/2)$ is obtained for a filter with an $R(e^{j\omega})$ that has sign changes. In a second step a new value for $a(0)$ is chosen as the original value obtained for $a(0)$ minus the minimum value of $Re^{j\omega}$. In this way the new set of coefficients defines a true linear phase filter. This method gives satisfactory results for low-pass, high-pass and bandstop filters, as well as for bandpass filters centred around $f_s/4$, where f_s is the sampling frequency.

4 Examples comparing linear phase with true linear phase filters

The examples presented in this section compare the behaviour and the characteristics of linear with true linear phase filters.

(*a*) In this example a simple waveform is generated and filtered by a linear phase filter. The same waveform is also filtered by a true linear phase filter having exactly the same amplitude response. The linear phase filter has an impulse response

$$h(n) = \begin{cases} 1 & n = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

This is a simple low-pass filter with an even-symmetric impulse response having integer coefficients. Its transfer function is

$$H(z) = \frac{z^4 + z^3 + z^2 + z + 1}{z^4} = \frac{z^5 - 1}{z^4(z-1)}$$

and the frequency response is $e^{-2j\omega}(\sin(5\omega/2))/\sin(\omega/2)$. Its magnitude as a function of ω is shown in Fig. 1*a*. The phase response is piecewise linear having discontinuities of size π (Fig. 1*a*) because $R(e^{j\omega}) = (\sin(5\omega/2))/\sin(\omega/2)$ changes sign at $\omega_1 = 4\pi/10$ and $\omega_2 = 8\pi/10$. The signal to be filtered is (Fig. 1*e*)

$$x(n) = \sin\left(\frac{2\pi}{10}n\right) + \sin\left(\frac{6\pi}{10}n\right),\ n \in Z$$

It is a superposition of two sinusoids, one at an angular frequency of $2\pi/10$ and the other $6\pi/10$. Note that the filter's phase response has a discontinuity at $\omega = 4\pi/10$, between the frequencies of the two sinusoids. The output signal is a very distorted version of the input (Fig. 1*f*). However, if the filter were a *true linear phase filter* (Fig. 1*b*), its output (Fig. 1*g*) would reflect the effects of the magnitude response only, without exhibiting distortions caused by the phase response. The waveform produced by the linear phase filter (Fig. 1*f*) is quite different from the ideal output

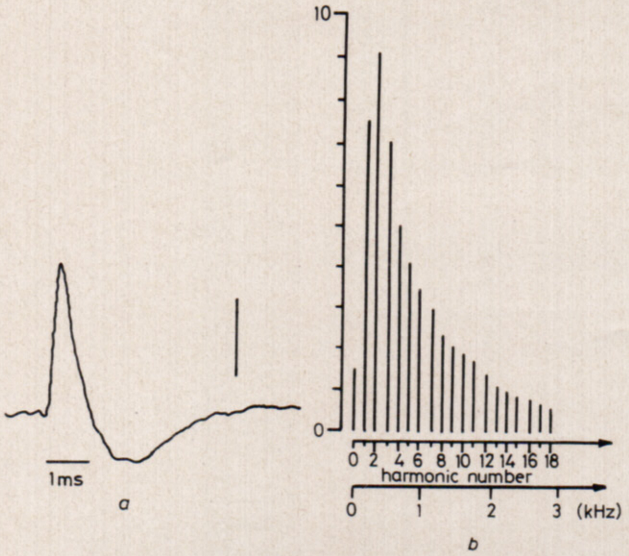

Fig. 2 *Motor unit action potential (MUAP); (a) MUAP obtained by superposition of DC + 18 harmonics obtained computing the FFT of the original MUAP; (b) absolute value (linear scale, arbitrary units) of the Fourier series components of the MUAP shown in (a). The abscissae in (b) show calibrations either in harmonic number or in frequency (kHz). The original MUAP was sampled at 20 kHz*

(Fig. 1*g*), the cause being the constant phase difference of π between the linear phase and the true linear phase filters for $\omega \in [4\pi/10, 8\pi/10)$. The same effects on the output waveforms may be observed when comparing the two noncausal filters in Figs. 1*c* and 1*d*. The noncausal filter outputs are similar to Figs. 1*f* and 1*g* except for the lack of delay with respect to the input signal.

The next three examples show the effects of FIR filtering on a bioelectrical signal: a motor unit action potential (MUAP). MUAPs were recorded (50–5000 Hz) using a concentric needle steel electrode inserted in the biceps brachii of a human subject. After A/D conversion, a 128-point FFT was obtained from a selected MUAP. The first 18 harmonics (cosine + sine) plus the DC level were kept. The signal obtained from the sum of the DC and the 18 harmonics will also be called the MUAP and is shown in Fig. 2*a*. The magnitudes of the Fourier series coefficients of this MUAP (i.e. the DC + 18 harmonics in the FFT) are shown in Fig. 2*b*. The filtering of this MUAP was achieved by multiplying each of its components by the desired filter's frequency response at the component's frequency. The superposition of these partial results constituted the filter's output. This procedure was followed because it was desired to compare the outputs of linear phase and true linear phase filters *having exactly the same magnitude response*, differing only in their phase responses.

(*b*) Nonrecursive notch FIR filters with very few integer coefficients are sometimes used in real-time filtering to

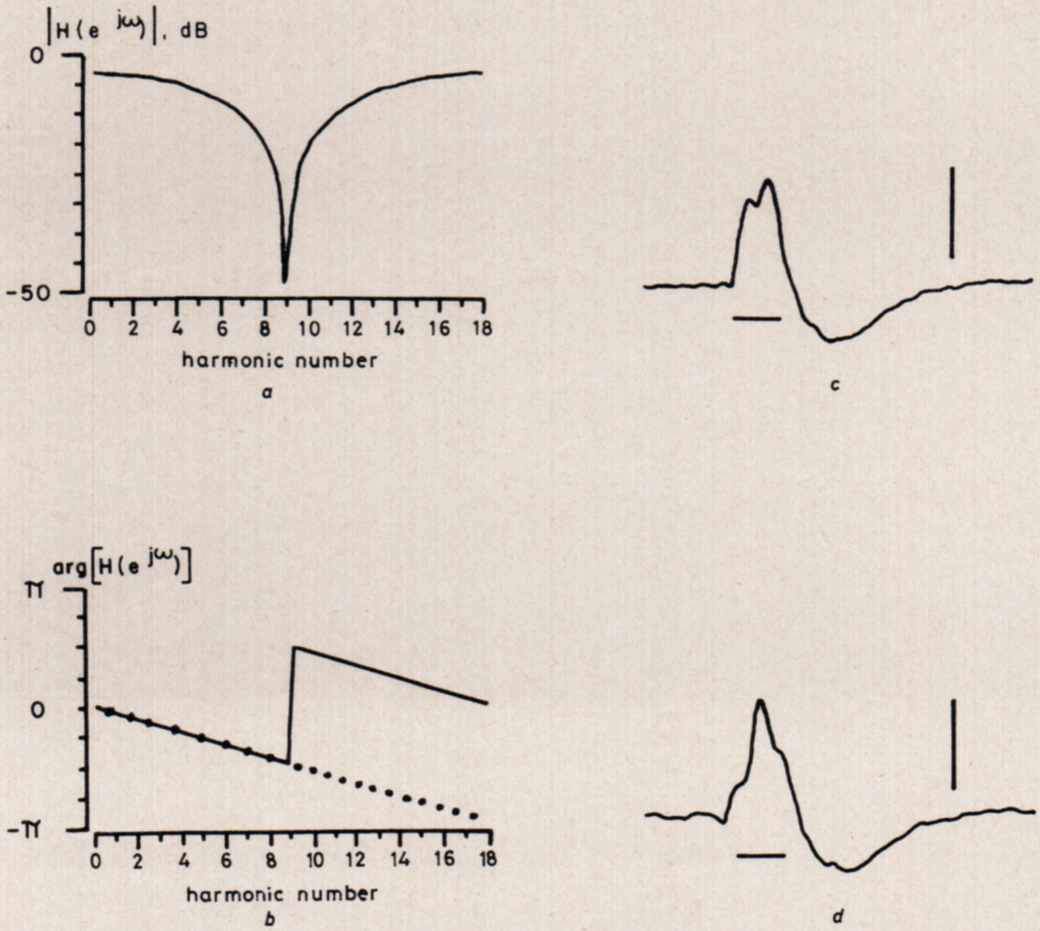

Fig. 3 *Effect of notch filtering on the MUAP of Fig. 2a. (a) Magnitude response of linear phase FIR filter* $H(z) = 1 + z^{-2}$ *and of corresponding true linear phase FIR filter; (b) phase response of the linear phase filter (solid line) and of the true linear phase version (dotted line); (c)* output *from linear phase filter; (d) output from true linear phase filter. Abscissae in (a) and (b) are calibrated according to the harmonics composing the input MUAP. Calibrations in (c) and (d) are the same, being also equal to that in Fig. 2a*

eliminate sinusoidal interference. A simple linear phase notch filter will be examined. Its impulse response is

$$h(n) = \begin{cases} 1 & n = 0 \\ 1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

and its transfer function is given by $H(z) = (z^2 + 1)/z^2$. Figs. 3*a* and 3*b* show the magnitude and the phase responses, respectively, of the notch filter described above. The abscissae show where the different harmonics that compose the MUAP fall. When the MUAP is passed through this filter the output is as seen in Fig. 3*c*. However, if the filter had true linear phase characteristics (the phase response in Fig. 3*b* would continue at the dotted lines after abscissa 9) the filtered MUAP would look like Fig. 3*d*. The difference between the two waveforms is dramatic; the linear phase filter output (Fig. 3*c*) exhibits distortions (e.g. an indentation near the peak) that could lead to deceiving interpretations. For example, the waveform of Fig. 3*c* could be interpreted as the superposition of two action potentials from different (populations of) excitable cells.

(*c*) In this example the same MUAP of Fig. 2*a* is low-pass filtered. The filter's impulse response is

$$h(n) = \begin{cases} 1 & n = 0, 1, \ldots, 6 \\ 0 & \text{otherwise} \end{cases}$$

The magnitude curve is remindful of that in Fig. 1*a* (except that here there are more humps), with the first dip chosen to occur at the 7th harmonic component of the MUAP. The phase curve has switchings at the 7th and 14th harmonics of the MUAP. The MUAP appears at the output of this linear phase filter as shown in Fig. 4*a*. If the filter had exactly the same magnitude response but no phase jumps (i.e. a true linear phase filter) the output would look like Fig. 4*b*. Again the two waveforms are rather dissimilar, the first

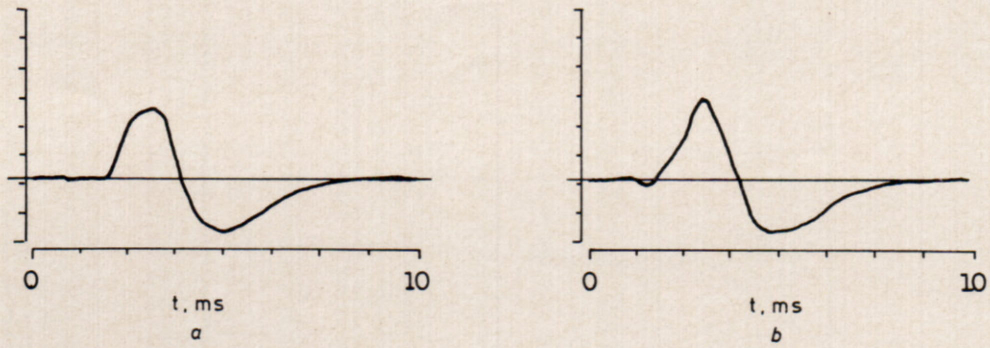

Fig. 4 *(a) Output from linear phase FIR filter $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}$ when MUAP was applied at the input; (b) output from true linear phase filter having the same magnitude response and the same input signal as in (a). Ordinates and abscissae in (a) and (b) are the same*

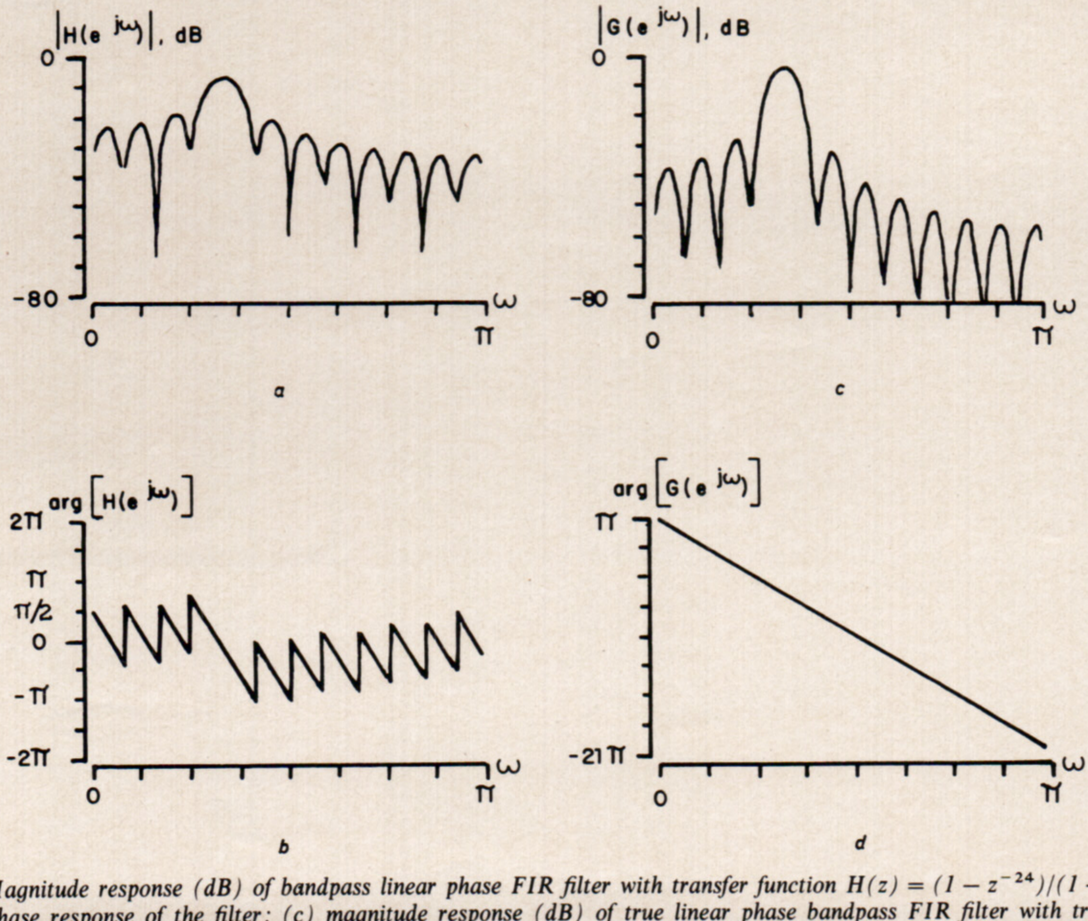

Fig. 5 *(a) Magnitude response (dB) of bandpass linear phase FIR filter with transfer function $H(z) = (1 - z^{-24})/(1 - z^{-1} + z^{-2})$; (b) phase response of the filter; (c) magnitude response (dB) of true linear phase bandpass FIR filter with transfer function $G(z) = (1 - z^{-24})^2/(1 - z^{-1} + z^{-2})^2$; (d) phase response of the true linear phase filter*

having an abnormally wide peak which lends it a trapezoidal shape whereas the second has a more triangular form.

A waveform filtered by a linear phase FIR filter with poorly attenuated stopbands (e.g. Fig. 4*a*) may have one or several parameters adversely affected by a piecewise linear phase response. If pulse amplitude, duration, risetime or latency with respect to a stimulus pulse (as in evoked potentials) are to be measured accurately then a true linear phase filter is highly preferable.

(*d*) A first example of the integer coefficient design presented in Section 3.1 will be a simple notch filter used for real-time mains interference rejection. Schluter (1981) implemented the following notch filter for a bedside arrhythmia monitor: $H(z) = 1 + z^{-3}$. This filter has a π phase jump at $\omega = \pi/3$. Therefore signal components from $\pi/3$ to π will be filtered with a sign inversion. However, if a true linear phase filter were desired, the following could have been used: $1 + 2z^{-3} + z^{-6} = (1 + z^{-3})^2$. This new filter would involve two extra arithmetic operations: a multiplication by 2 (i.e. a binary shift) and an addition. Generally speaking, the small increase in the computation time required by a true linear phase design is outweighed by the considerable improvement in the filtering performance.

(*e*) Lynn's (1977, section 3.2, page 536) bandpass filter $H(z) = (1 - z^{-24})/(1 - z^{-1} + z^{-2})$ has its first sidelobes approximately 13 dB below the main lobe, as can be seen from Fig. 5*a*. Its phase response shows several jumps of size π and is described by $(\pi/2) - 11\omega$ for small ω (Fig. 5*b*). The filter can be expected to introduce considerable distortions not only due to the π phase difference in frequency regions corresponding to poorly attenuated sidelobes but also due to the $\pi/2$ constant term. In other words, there would be phase distortion even if the input signal's energy were concentrated only within the main lobe. A great improvement in performance can be obtained if the poles and zeros of the transfer function $H(z)$ given are taken with *even* multiplicities as discussed in Section 3.1. Figs. 5*c* and 5*d* show the magnitude and phase responses, respectively, for $G(z) = (1 - z^{-24})^2/(1 - z^{-1} + z^{-2})^2$. The sidelobes decrease, as already pointed out by Lynn (1977), but an equally important feature is that the new phase response has no phase jumps nor any $\pm\pi/2$ term. This true linear phase filter will not introduce any phase distortion whatsoever.

(*f*) This example illustrates the second type of window design proposed in Section 3.2 for true linear phase filters. A low-order low-pass linear phase FIR filter was designed using a simple interactive program based on the window technique. The rectangular window was used to minimise the width of the transition band. The order of the filter was chosen to be 6 so that N is odd (= 7). The resulting frequency response is shown in Fig. 6*a*, with the amplitude response a solid line and the phase response in broken lines. The dotted lines indicate $R(e^{j\omega})$ wherever it is different from $|H(e^{j\omega})|$. The same magnitude response is plotted in dB in Fig. 6*b*. The program provided the following symmetric impulse response coefficients (keeping only five digits for the mantissa):

$$h(0) = 0{\cdot}08584 = h(6)$$

$$h(1) = 0{\cdot}09355 = h(5)$$

$$h(2) = 0{\cdot}09836 = h(4)$$

$$h(3) = 0{\cdot}10000$$

As the resulting attenuation of the sidelobes is rather small the phase jumps may lead to hazardous distortions. The effect of increasing $h(3)$ (i.e. $a(0)$ in eqn. 6) is that of raising $R(e^{j\omega})$. For large values of $h(3)$ $R(e^{j\omega})$ will be completely above the horizontal axis but the ideal condition is when it just touches it (Fig. 6*c*). This condition is met when $h(3) = 0{\cdot}23242$. The resulting true linear phase filter has an acceptable magnitude response (Fig. 6*d*, solid line) showing a widening of the passband and a decrease of about 3 dB in the minimum attenuation at the stopband. If the user wants to obtain a narrower passband he should choose a larger N and repeat the procedure. If smaller sidelobes are desired a different window should be used.

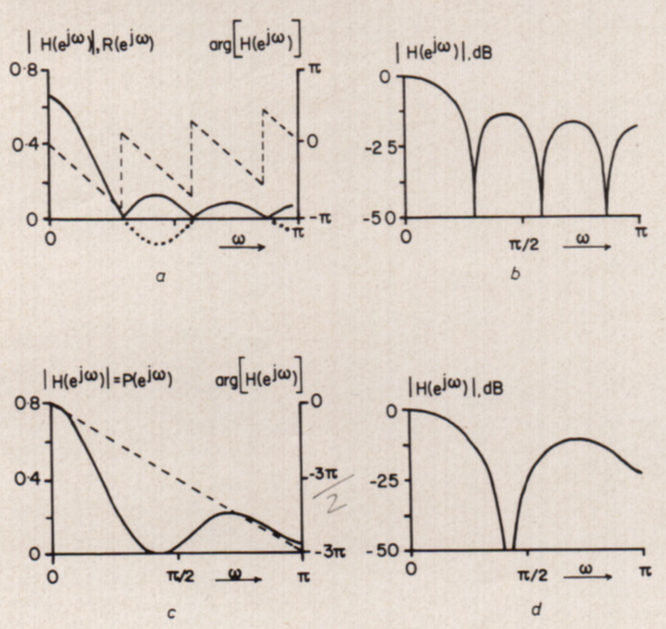

Fig. 6 *(a) Magnitude (solid line) and phase response (broken line) of linear phase FIR filter with impulse response coefficients h(0) = h(6) = 0·08584, h(1) = h(5) = 0·09355, h(2) = h(4) = 0·09836, h(3) = 0·10000. The dotted line indicates* $R(e^{j\omega})$ *wherever it is different from* $|H(e^{j\omega})|$*; (b) magnitude response in dB of the filter in (a); (c) magnitude (solid line) and phase response (broken line) of true linear phase filter obtained by increasing h(3) to the value 0·23242. The magnitude curve is a shifted up version of* $R(e^{j\omega})$ *in (a); (d) magnitude response in dB of the filter in (c)*

Acknowledgments—This work was supported by a grant from FAPESP (83/0433-7). The author thanks Professor C. Timo-Iaria for his help in the EMG recording and Professors J. J. Angerami and N. Alens for reading the manuscript.

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